Sheet 4

System of First Order and Higher Order Ordinary Differential Equations

1- Solve the following system of simultaneous equation with associated initial conditions using **Heun's method**.

$$\frac{dy}{dx} = y - z, \qquad y(0) = 0$$

$$\frac{dz}{dx} = -y + z, \quad z(0) = 1$$

in the range $0 \le x \le 1$ with step size of 0.5.

2- Solve $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} - 2xy = 0;\ y(0) = 1, \frac{dy}{dx}(0) = 0$ for y(0.1) using **Heun's method**. with step size of 0.05.

3. The motion of a damped spring-mass system as shown in the Figure is described by the following ordinary differential equation

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0$$

where

x : displacement from equilibrium position (m),

t : time (s),

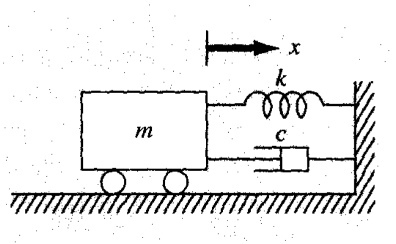
m: mass = 10-kg, and

c : the damping coefficient=5 (N • s/m).

k: the spring constant = 40 N/m.

The initial velocity is zero, and the initial displacement x = 1 m.

Solve this equation using **Heun's method** over the time period 0 <= t<=10 s. consider time step of 5 sec.



4. Solve the following set of differential equations using **Runge-Kutta Method**, assuming that at x = 0, $y_1 = 3$, and $y_2 = 5$. Integrate to x = 2 with a step size of 1.

$$\frac{dy_1}{dx} = -0.5y_1$$

$$\frac{dy_2}{dx} = 4 - 0.1y_1 - 0.3y_2$$

5. Using **Runge-Kutta** method, solve the following initial-value problems for the second-order ordinary differential equations for $0 \le x \le 1$, h = 0.5, y(0) = 0.5 and y'(0) = −0.5. by reducing the equations to a system of first-order ordinary differential equations

$$\frac{d^2y}{dx^2} = 3x\frac{dy}{dx} + 2xy - 3x - 2,$$

6. Use the **Runge-Kutta** method to solve

$$\frac{d^2y}{dt^2} - t + y = 0$$

where y(0) = 2 and y'(0) =0. Solve from x = 0 to 4 using h = 2.

1 - السؤال الاول والرابع محلولين حل نموذجى
2 - السؤال الثانى والخامس سيتم شرحهم فى السكشن
3 - السؤال الثالث والسادس سيحلهم الطالب ويقدمهم فى تقرير منظم فى الموعد الذى سيحدده المعيد
4 - فى حالة تقديم التقرير بعد الموعد المحدد فلن يقبل منه مهما كانت الاعذار ولن توضع له درجة

1- $Replace\ department\ variables\ y\ by\ y_1 and, z\ by\ y_2 then$

$$\frac{dy_1}{dx} = y_1 - y_2, \qquad y_1(0) = 0$$

$$\frac{dy_2}{dx} = -y_1 + y_2, \quad y_2(0) = 1$$

$Given$

$$f_1(x, y_1, y_2) = y_1 - y_2$$
$$f_2(x, y_1, y_2) = -y_1 + y_2$$
$$x(0) = 0$$
$$y_1(0) = 0$$
$$y_2(0) = 1$$
$$x_f = 1$$
$$h = 0.5$$

$Solution$

$y_1(1), y_2(1)$

$$x_1 = x_0 + h = 0 + 0.5 = 0.5$$

$$y_1^e(1) = y_1(0) + hf_1\big(x(0), y_1(0), y_2(0)\big) = 0 + 0.5 * f_1(0, 0, 1) = -0.500$$

$$y_2^e(1) = y_2(0) + hf_2\big(x(0), y_1(0), y_2(0)\big) = 1 + 0.5 * f_2(0, 0, 1) = 1.500$$

$$y_1(1) = y_1(0) + h\frac{f_1\big(x(0), y_1(0), y_2(0)\big) + f_1\big(x(1), y_1^e(1), y_2^e(1)\big)}{2}$$

$$= 0 + 0.5\frac{f_1(0, 0, 1) + f_1(0.5, -0.500, 1.500)}{2} = -0.750$$

$$y_2(1) = y_2(0) + h\frac{f_2\big(x(0), y_1(0), y_2(0)\big) + f_2\big(x(1), y_1^e(1), y_2^e(1)\big)}{2}$$

$$= 1 + 0.5\frac{f_2(0, 0, 1) + f_2(0.5, -0.500, 1.500)}{2} = 1.750$$

$y_1(2), y_2(2)$

$$x_2 = x_1 + h = 0.5 + 0.5 = 1$$

$$y_1^e(2) = y_1(1) + hf_1\big(x(1), y_1(1), y_2(1)\big) = -0.750 + 0.5 * f_1(0.5, -0.750, 1.750) = -2$$

$$y_2^e(2) = y_2(1) + hf_2\big(x(1), y_1(1), y_2(1)\big) = 1.750 + 0.5 * f_2(0.5, -0.750, 1.750) = 3$$

$$y_1(2) = y_1(1) + h\frac{f_1\big(x(1), y_1(1), y_2(1)\big) + f_1\big(x(2), y_1^e(2), y_2^e(2)\big)}{2}$$

$$= -0.750 + 0.5\frac{f_1(0.5, -0.750, 1.750) + f_1(1, -2, 3)}{2} = -2.625$$

$$y_2(2) = y_2(1) + h\frac{f_2\big(x(1), y_1(1), y_2(1)\big) + f_2\big(x(2), y_1^e(2), y_2^e(2)\big)}{2}$$

$$= 1.750 + 0.5\frac{f_2(0.5, -0.750, 1.750) + f_2(1, -2, 3)}{2} = 3.625$$

i	x	y	z
0	0	0	1
1	0.5	-0.750	1.750
2	1	-2.625	3.625

$4\text{-}Given$
$f_1(x\,, y_1, y_2) = \; -0.5y_1$
$f_2(x\,, y_1, y_2) = \; 4 - 0.1y1 - 0.3y_2$
$x(0) = \; 0$
$y_1(0) = \; 3$
$y_2(0) = \; 5$
$x_f \; = \; 2$
$h \; = \; 1$

$Solution$
$y_1(1), y_{2(1)}$
$x_1 \; = x_0 + h \; = 0 + 1 \; = 1$
$k_{1,1} = \; f_1\big(x(0), y_1(0), y_2(0)\big) \; = \; f_1(0\,,3\,,5\,) = -1.500$
$k_{1,2} = \; f_2\big(x(0), y_1(0), y_2(0)\big) \; = \; f_2(0\,,3\,,5\,) = 2.200$
$k_{2,1} = \; f_1\left(x(0) + \frac{h}{2}, y_1(0) + h\frac{k_{1,1}}{2}, y_2(0) + h\frac{k_{1,2}}{2}\right) = \; f_1(0.500\,,2.250\,,6.100) = -1.125$
$k_{2,2} = \; f_2\left(x(0) + \frac{h}{2}, y_1(0) + h\frac{k_{1,1}}{2}, y_2(0) + h\frac{k_{1,2}}{2}\right) = \; f_2(0.500\,,2.250\,,6.100) = 1.945$
$k_{3,1} = \; f_1\left(x(0) + \frac{h}{2}, y_1(0) + h\frac{k_{2,1}}{2}, y_2(0) + h\frac{k_{2,2}}{2}\right) = \; f_1(0.500\,,2.438\,,5.972) = -1.219$
$k_{3,2} = \; f_2\left(x(0) + \frac{h}{2}, y_1(0) + h\frac{k_{2,1}}{2}, y_2(0) + h\frac{k_{2,2}}{2}\right) = \; f_2(0.500\,,2.438\,,5.972) = 1.965$
$k_{4,1} = \; f_1\big(x(0) + h\,, y_1(0) + h\, k_{3,1}, y_2(0) + h\, k_{3,2}\big) \; = \; f_1(1\,,1.781\,,6964) = -0.891$
$k_{4,2} = \; f_2\big(x(0) + h\,, y_1(0) + h\, k_{3,1}, y_2(0) + h\, k_{3,2}\big) \; = \; f_2(1\,,1.781\,,6.964) = 1.733$

$$y_1(1) = y_1(0) + h * \frac{k_{1,1} + 2 * k_{2,1} + 2 * k_{3,1} + k_{4,1}}{6}$$
$$= 3 + 1 * \frac{(-1.500) + 2 * (-1.125) + 2 * (-1.219) + (-0.891)}{6} = 1.820$$
$$y_2(1) = y_2(0) + h * \frac{k_{1,2} + 2 * k_{2,2} + 2 * k_{3,2} + k_{4,2}}{6}$$
$$= 5 + 1 * \frac{(2.200) + 2 * (1.945) + 2 * (1.965) + (1.733)}{6} = 6.959$$

$\boldsymbol{y_1(2), y_{2(2)}}$
$x_2 = x_1 + h \; = 1 + 1 \; = 2$
$k_{1,1} = \; f_1\big(x(1), y_1(1), y_2(1)\big) \; = \; f_1(1\,,1.820\,,6.959\,) = -0.910$
$k_{1,2} = \; f_2\big(x(1), y_1(1), y_2(1)\big) \; = \; f_2(1\,,1.820\,,6.959\,) = 1.730$
$k_{2,1} = \; f_1\left(x(1) + \frac{h}{2}, y_1(1) + h\frac{k_{1,1}}{2}, y_2(1) + h\frac{k_{1,2}}{2}\right) = \; f_1(1.500\,,1.365\,,7.824) = -0.683$
$k_{2,2} = \; f_2\left(x(1) + \frac{h}{2}, y_1(1) + h\frac{k_{1,1}}{2}, y_2(1) + h\frac{k_{1,2}}{2}\right) = \; f_2(1.500\,,1.365\,,7.824) = 1.516$
$k_{3,1} = \; f_1\left(x(1) + \frac{h}{2}, y_1(1) + h\frac{k_{2,1}}{2}, y_2(1) + h\frac{k_{2,2}}{2}\right) = \; f_1(1.500\,,1.479\,,7.717) = -0.740$
$k_{3,2} = \; f_2\left(x(1) + \frac{h}{2}, y_1(1) + h\frac{k_{2,1}}{2}, y_2(1) + h\frac{k_{2,2}}{2}\right) = \; f_2(1.500\,,1.479\,,7.717) = 1.537$
$k\_4,1 \; = \; f\_1\,(x(1) + h\,, y\_1\,(1) + h\, k\_3,1\,, y\_2\,(1) + h\, k\_3,2) \; = \; f\_1\,(2\,,1.081\,,8.496) \; = -0.540$
$k\_4,2 \; = \; f\_2\,(x(1) + h\,, y\_1\,(1) + h\, k\_3,1\,, y\_2\,(1) + h\, k\_3,2) \; = \; f\_2\,(2\,,1.081\,,8.496) \; = 1.343$

$$y_1(2) = y_1(1) + h * \frac{k_{1,1} + 2 * k_{2,1} + 2 * k_{3,1} + k_{4,1}}{6}$$
$$= 1.820 + 1 * \frac{(-0.910) + 2 * (-0.683) + 2 * (-0.740) + (-0.540)}{6} = 1.105$$
$$y_2(2) = y_2(1) + h * \frac{k_{1,2} + 2 * k_{2,2} + 2 * k_{3,2} + k_{4,2}}{6}$$

$$= 6.959 + 1 * \frac{(1.730) + 2 * (1.516) + 2 * (1.537) + (1.343)}{6} = 8.489$$

i	x	y_1(i)	y_2(i)
0	0	3	5
1	1	1.820	6.959
2	2	1.105	8.489